



Advances in Southern Ocean tide modeling

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Abstract

Tides in the polar region play a major role in the dynamics of sea ice and floating glacial ice shelves. Existing ocean tide models are much less accurate in coastal and shallow seas and polar oceans than other ocean areas. In particular, ocean tides are largely unknown in parts of polar oceans that are covered by permanent or seasonal sea ice and in regions that are beyond the coverage of the TOPEX/POSEIDON (T/P). We conducted a periodogram analysis using simulated satellite altimeter data (T/P or JASON, ERS-2 or ENVISAT) at single and dual satellite crossover locations. Our results indicate that the TOPEX/ERS-2 dual satellite crossovers show considerable improvements to mitigate tidal aliasing and are, therefore, preferable for tidal modeling. Empirical ocean tide models have been determined using the T/P and ERS-2 altimetry at crossovers in the Southern Ocean below 55°S and employing the response method. Evaluations using in situ pelagic tidal constants and altimeter data indicate that the resulting empirical tide models are as good as the selected contemporary models (NAO.99b, TPXO.6.2, and CATS02.01) in the Southern Ocean. Solutions for the M_2 , O_1 , N_2 , and Q_1 tides that are obtained from the ERS-2 data only show a reasonable accuracy, indicating that extensions of the tide modeling domain beyond the T/P coverage are feasible.

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1. Improved temporal-sampling at crossover locations

Smith (1999) points out the advantage of using single satellite crossover data in tidal analysis: it is easier to separate tidal constituents whose aliased frequencies are close to each other due to the phase difference between data of crossing tracks.

To study the feasibility of improving ocean tide modeling by the use of multiple satellite altimeter data, we performed a frequency analysis using simulated T/P and ERS altimeter data that are sampled at single and dual satellite crossovers. The numerical tool we used is the so-called periodogram, whose definition is generalized to include the time series data of arbitrary sampling intervals (Mautz, 2002). A global optimization method of least squares with an interval search algorithm employed in this tool makes it possible to analyze the spectra of time series data sampled at uneven sampling rates. This technique estimates the Lomb–Scargle periodogram of satellite altimetry sea surface heights sampled at a

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Table 1

Periodogram of simulated data at a T/P single satellite crossover point (Gaussian white noise of 3 cm rms added without a linear trend)

Harmonics	Frequency (cycle/day)		Estimated frequency (cycle/day)	True amplitude (cm)	Vector difference (cm)
	True	Aliased			
M ₂	1.932274		1.932273	16.6	0.18
S ₂	2.000000	0.017024	0.017012	5.0	4.05
K ₁	1.002738		1.002734	11.1	0.34
O ₁	0.929536		0.929538	10.9	0.28
N ₂	1.895982	0.020191	1.432113	4.0	1.19
P ₁	0.997262		0.997243	3.8	0.81
K ₂	2.005476	0.011548	0.616649	1.4	0.87
Q ₁	0.893244	0.014417	0.417809	2.6	5.09
Annual	0.002738		0.002739	8.5	0.20
Root sum of squares				25.57	6.74

crossover location having either mixed sampling periods (dual satellite crossovers) or a single sampling period with samples taken at different phases (single satellite crossovers).

Simulated data of an approximately 8-year long data span were generated for both satellites. Mautz's periodogram software was used to search for 12 highest peaks within a frequency band that goes up to 2.19 cycles per day. The results shown in Table 1 demonstrate that appropriate harmonic analyses for single satellite crossover data should be able to accurately identify the harmonic constants (phase and amplitude) that correspond to true non-aliased frequencies of dominant short-period tides depending on their amplitudes and the data noise strength. From the noise-free simulated data, a perfect recovery of true harmonic constants for all frequency contents was obtained by the search method of Mautz's periodogram software. The vector difference in Table 1 is the modulus of vector difference between true and estimated parameters. The vector is defined as a complex exponential function of the imaginary phase angle, multiplied by the real amplitude (Ray, 1993).

The periodogram of evenly sampled time series data is a periodic function of frequency. Thus, a temporal-sampling at fixed intervals results in the aliasing problem if the original data before temporal-sampling contain harmonics whose frequencies are higher than one-half of the Nyquist frequency because of the frequency-folding phenomenon. This means that any tidal analysis results using altimeter data along repeating tracks can only estimate the constants of aliased harmonics. However, the periodograms of unevenly sampled data are not periodic in frequency, so that they have a reduced aliasing effect (Scargle, 1982). Mautz's global optimization algorithm for a sum of squared fit residuals (a negative Lomb–Scargle periodogram) is able to locate many of dominant local peaks in the periodogram at true tidal frequencies primarily because the periodogram is not a periodic function of frequency. Thus, the numerical test results of frequency analysis shown in Tables 1 and 2 (M₂, K₁, O₁, and P₁ tides) clearly indicate the advantage of using the altimeter data sampled at single and dual satellite crossover locations, respectively, to mitigate the

Table 2

Periodogram of simulated data at a T/P and ERS-2 dual satellite crossover point (Gaussian white noise of 3 cm rms added to T/P data and that of 8 cm rms added to ERS-2 data without a linear trend)

Harmonics	True frequency (cycle/day)	Estimated frequency (cycle/day)	True amplitude (cm)	Vector difference (cm)
M ₂	1.932274	1.932279	16.6	0.59
S ₂	2.000000	2.000012	5.0	0.48
K ₁	1.002738	1.002738	11.1	0.11
O ₁	0.929536	0.929541	10.9	0.36
N ₂	1.895982	1.895984	4.0	0.23
P ₁	0.997262	0.291297	3.8	0.30
K ₂	2.005476	1.020117	1.4	0.22
Q ₁	0.893244	0.316972	2.6	5.73
Annual	0.002738	0.002731	8.5	1.10
Root sum of squares			25.57	5.91

aliasing problem in tidal analyses. Non-aliased frequencies of S_2 , N_2 , K_2 , and Q_1 tides, however, are not successfully recoverable from the T/P single satellite crossover data nor are those of P_1 , K_2 , and Q_1 tides from the T/P and ERS-2 dual satellite crossover data. Incidentally, these S_2 , N_2 , P_1 , K_2 , and Q_1 tides have amplitudes that are close or lower than the 3 cm TOPEX data noise level. It can be concluded that the T/P and ERS-2 dual satellite crossover case is better than the T/P single satellite crossover case, at least in terms of the root sum of squared vector differences.

The same periodogram tool was applied to actual altimeter data at several dual satellite crossover points of T/P and ERS-2 in the Southern Ocean (Wang, 2004). The eight dominant short-period tidal constituents except K_2 and Q_1 were successfully identified confirming the existence of non-aliased harmonic information of short-period ocean tides in the multiple satellite altimeter data. The tidal constituents identifiable from actual altimeter data in the test area have the strongest amplitudes in the order of M_2 , K_1 , O_1 , S_2 , N_2 , and P_1 . For a few crossover points, however, S_2 , N_2 , nor P_1 tides were not identifiable using this technique.

2. Empirical tidal solution in the Southern Ocean

In this study, a point-wise tidal analysis was performed using the TOPEX and ERS-2 altimeter data at crossover locations in the Southern Ocean below the 55°S latitude. A modified version of Ole Andersen's (1994) orthotide tidal analysis software was used to solve for eight dominant short-period tides along with four long-period tides, S_a , S_{sa} , M_m , and M_f . The simultaneous solution of these long-period tides is intended mostly to absorb oceanographic signals. In addition, a bias for each of crossing tracks was included in the tidal solution.

The sea surface height (SSH) anomaly data of TOPEX cycles 4–364 and ERS-2 cycles 1–79 were used in the solution. Each of eight dominant short-period tides was then interpolated and saved on a regular grid of $0.25^\circ \times 0.25^\circ$ to be evaluated against the in situ data in the Southern Ocean below the 58°S parallel. Two sets of ground truth data that had been provided by Richard D. Ray in 1999 were used for the model evaluation of this study. The pelagic tidal constants are available at 102 tide gauge stations in the first data set and the coastal data set contains tidal constants at 739 sites. Of these stations, 3 pelagic sites and 13 coastal sites in the Southern Ocean were selected.

Three empirical tide models of this study, corresponding to the TOPEX single satellite, the TOPEX/ERS-2 dual satellite, or the ERS-2 single satellite crossover data, were compared with the tidal constants at these selected tide gauge sites. Table 3 shows the rms differences of tidal constants of these models from the in situ data at three pelagic sites (depicted with white stars in Fig. 1) by constituents and also for all of seven constituents that are combined in a root sum of squares (RSS) sense. Three contemporary ocean tide models were also included in the ground truth comparison (Egbert and Erofeeva, 2002; Matsumoto et al., 2000; Padman et al., 2002). Two models of this study that include the TOPEX altimeter data have the agreements with in situ data that are similar to those of the NAO.99b and TPXO.6.2 models. Our tidal solution based on ERS-2 data only, however, shows a poor comparison, which can be expected from the tidal aliasing of the 35-day repeat cycle. The poor tidal solution should be due to the fact that S_2 , K_1 , P_1 , and K_2 tides are not separable from bias, S_a , S_{sa} , and S_{sa} , respectively, when ERS-2 data are used only. Even the response method using the single satellite crossover data does not appear to be able to improve this severe aliasing.

To test the improved tidal sampling at crossover locations that was observable from our simulated altimeter data analysis, we produced the alternative empirical models using the TOPEX data of only one of two crossing tracks at

Table 3
Ground truth comparison of ocean tide models at three pelagic tide gauge stations in the Southern Ocean below 58°S (in cm)

Model	M_2	S_2	K_1	O_1	N_2	P_1	K_2	RSS
NAO.99b	1.90	0.99	1.46	1.01	0.36	0.43	0.38	2.87
TPXO.6.2	1.59	0.99	1.21	0.74	0.24	0.36	0.31	2.41
CATS02.01	2.12	1.29	1.11	0.92	0.44	0.29	0.37	2.95
TOPEX, this study	1.51	0.87	1.33	0.84	0.28	0.43	0.35	2.43
TOPEX/ERS-2, this study	1.69	1.09	1.01	0.87	0.39	0.29	0.38	2.49
ERS-2, this study	2.12	–	–	1.25	0.51	–	–	–

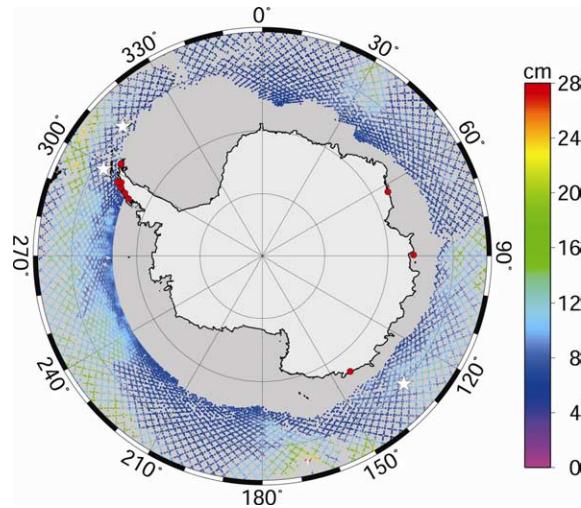


Fig. 1. Standard deviations of residual TOPEX SSH anomaly at the dual satellite crossover locations of TOPEX and ERS-2 satellites after removing tidal height predictions of the TOPEX/ERS-2 empirical model and location of tide gauge sites used as the ground truth (3 pelagic sites shown as white stars and 13 coastal sites as red dots). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

crossover points. That is, we discarded one of two crossing tracks at each crossover location to get these alternative models. We did obtain the improved comparison for the TOPEX/ERS-2 dual satellite crossover data (2.49 cm RSS in Table 3) compared with 2.57 cm RSS of the corresponding alternative model, for which no ERS-2 data were included. However, no such improved tidal information seems to exist in the TOPEX single satellite crossover data considering the RSS differences (2.43 cm RSS in Table 3 compared with 2.37 cm RSS of the alternative model). These results along with those from our periodogram analysis suggest that the dual satellite crossover data are better than single satellite crossover data in improving tidal sampling of satellite altimetry. It should be pointed out that one more advantage of the dual satellite crossovers is that they have more crossover locations than the corresponding single satellite crossovers. This abundance comes from the fact that the dual satellite crossovers are formed by more combinations of crossing geometry.

The empirical models of this study that include the TOPEX data show above-average performances in Table 3. However, the TOPEX single satellite crossover model shares the worst P_1 comparison of 0.43 cm with the NAO.99b model and the dual satellite crossover model shows one of the poorest K_2 comparisons. Table 4 summarizes standard deviations of residual SSH anomaly after removing tidal height predictions of ocean tide models at the TOPEX/ERS-2 dual satellite crossover points. The standard deviations of difference between TOPEX SSH anomaly and the TOPEX/ERS-2 model prediction of this study are shown in Fig. 1. NAO.99b model and the TPXO.6.2 model show the best agreements with altimeter data. The ground truth comparisons of models at 13 coastal sites in Table 5 show that no models have results as good as those of 3 pelagic sites. The huge data gaps that can be seen in Fig. 1 between the available TOPEX data points and the coastlines of the Antarctica should explain the poor comparison results in Table 5 very well.

Table 4

Standard deviations of residual SSH anomaly of ocean tide models at 13,760 TOPEX/ERS-2 dual satellite crossovers in the Southern Ocean below 58°S (in cm)

Altimeter data	NAO.99b	TPXO.6.2	TOPEX/ERS-2, this study	CATS02.01
TOPEX	8.4	8.6	8.8	9.0
ERS2	10.0	10.1	10.4	10.6

Table 5

Ground truth comparison of ocean tide models at 13 coastal tide gauge stations in the Southern Ocean below 58°S (in cm)

Model	M ₂	S ₂	K ₁	O ₁	N ₂	P ₁	K ₂	Q ₁	RSS
NAO.99b	5.85	5.65	11.07	12.78	1.96	4.05	1.87	2.43	19.54
TPXO.6.2	6.77	6.22	9.67	11.50	2.01	5.15	1.62	3.03	18.78
CATS02.01	6.86	6.52	9.59	11.51	2.20	3.64	1.87	2.61	18.50
TOPEX single, this study	9.15	7.63	9.93	11.41	2.11	3.45	2.16	2.14	19.91
TOPEX/ERS-2, this study	7.33	5.89	9.27	10.75	2.03	3.31	1.69	1.99	17.66
ERS-2 single, this study	8.00	—	—	10.99	1.84	—	—	1.80	—

3. Conclusions

The tidal aliasing problem means that the recovery of non-aliased harmonics of short-period tides is not possible using the altimeter data of a single satellite sampled on repeating tracks. However, a frequency analysis of radar altimeter data at single and dual satellite crossover points using the Lomb–Scargle periodogram reveals the existence of non-aliased harmonic information in the data for several short-period ocean tides.

An empirical tidal solution at the single and dual satellite crossover points of T/P and ERS-2 satellites in the Southern Ocean indicates that the agreement with the in situ tidal constants is as good as or better than three contemporary tide models. The empirical model based on the TOPEX/ERS-2 dual satellite crossover data shows an improvement over the case of the ERS-2 data stripped off at the same crossover locations. The model accuracy is about 2.5 cm for both the TOPEX single satellite crossovers and the TOPEX and ERS-2 dual satellite crossovers compared with the in situ data of seven short-period tides.

Both the ground truth evaluation of empirical model and the frequency analysis results of simulated data show that a better tidal sampling is obtainable at the dual satellite crossover points than at the single satellite crossovers. Thus, improvements of tidal solution at latitudes where no T/P or JASON data are available are anticipated by including altimeter data of more satellites, such as GFO and ENVISAT in the form of dual satellite crossover data. The ERS-2 only solutions for the M₂, O₁, N₂, and Q₁ tides show a reasonable accuracy although the over-parametrization caused by the tidal aliasing makes the tidal solutions using the ERS-2 single satellite crossover data ill-conditioned. This indicates that the extension of tide modeling domain beyond the T/P coverage is feasible if the dual satellite crossover data are used.

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